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Plane Strain Deformation in Generalized Thermoelastic Diffusion

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Abstract The present investigation is concerned with plane strain deformation in homogeneous isotropic generalized thermoelastic diffusion subjected to a normal force, thermal source, and chemical potential source. Laplace and Fourier transform techniques are employed to solve the problem. The integral transform have been inverted by using a numerical technique to obtain the displacements, stresses, temperature distribution, and chemical potential distribution. The numerical results of these quantities are illustrated graphically to depict the response of various sources in the theories of thermoelastic diffusion and thermoelasticity for a particular model. Some particular cases have been deduced from the present investigation.

Keywords Chemical potential source · Generalized thermoelastic diffusion · Laplace and Fourier transforms · Mechanical force · Thermal source

1 Introduction

The classical theory of dynamic thermoelasticity takes into account the coupling between temperature and strain fields. However, the dynamic coupled theory involves a contradiction that thermal waves propagate at an infinite velocity. The theory of generalized thermoelasticity has been developed in an attempt to eliminate the paradox of the infinite velocity of thermal propagation. Therefore, the generalized theory is the dynamic coupled thermoelasticity which includes the time needed for the acceleration of a thermal wave.

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There are two different theories of generalized thermoelasticty: the first is proposed by Lord and Shulman [1], and the second is proposed by Green and Lindsay [2]. The theory of Lord and Shulman (L–S theory) is based on the modified Fourier law of heat conduction and assumes one relaxation time whereas the theory of Green and Lindsay (G–L theory) uses a modified energy equation, constitutive equations and allows two relaxation times.

The study of diffusion phenomena is of great interest due to its many applications in geophysics and industrial applications. Diffusion can be defined as the random walk of an ensemble of particles from regions of high concentration to regions of lower concentration. In integrated circuit fabrication, diffusion is used to introduce dopants in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, to form integrated resistors, to form the source/drain regions in metal oxide semiconductor (MOS) transistors, and to dope poly-silicon gates in MOS transistors. The study of the phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits); the process of thermodiffusion can be used for more efficient extraction of oil from oil deposits.

Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion, and strain. Heat and mass exchange with the environment during thermodiffusion in an elastic solid. Using the coupled thermoelastic model, Nowacki [3–6] developed the theory of thermoelastic diffusion and discussed dynamical problems of diffusion in solids. Olesiak and Pyryev [7] discussed a coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. They studied the influences of cross effects arising from the coupling of the fields of temperature, mass diffusion, and strain. Due to these cross effects, the thermal excitation results in an additional mass concentration and the mass concentration generates the additional field of temperature. Genin and Xu [8] investigated a problem on thermoelastic plastic metals with mass diffusion.

Sherief et al. [9] developed the theory of generalized thermoelastic diffusion with one relaxation time, which allows finite speeds of propagation of waves. Sherief and Saleh [10] investigated a half-space problem in the theory of generalized thermoelastic diffusion with one relaxation time. Singh [11,12] discussed the reflection phenomena of waves from a free surface of an elastic solid with generalized thermodiffusion. Recently, Aouadi [13] studied thermoelastic–diffusion interactions in an infinitely long solid cylinder subjected to thermal shock on its surface with a permeating substance. Aouadi [14] investigated the problem of a thermoelastic half-space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time and with variable electrical and thermal conductivities.

The objective of the present investigation is to determine the components of displacement, stress, temperature distribution, and chemical potential distribution in an isotropic homogeneous elastic solid with generalized thermoelastic diffusion subjected to a normal force, thermal source, and chemical potential source.

2 Basic Equations

Following Lord and Shulman [1], Green and Lindsay [2], and Sherief et al. [9], the governing equations for an isotropic homogeneous elastic solid with generalized thermoelastic diffusion in the absence of body forces and heat sources include the constitutive relations,

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1(\theta + \tau_1\dot{\theta}) - \beta_2(C + \tau^1\dot{C})], \tag{1}$$

$$P = -\beta_2 e_{kk} + b(C + \tau^1\dot{C}) - a(\theta + \tau_1\dot{\theta}), \tag{2}$$

the equation of motion,

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1(\theta + \tau_1\dot{\theta})_{,i} - \beta_2(C + \tau^1\dot{C})_{,i} = \rho\ddot{u}_i, \tag{3}$$

the equation of heat conduction,

$$\rho C_E(\dot{\theta} + \tau_0\ddot{\theta}) + \beta_1 T_0(\dot{e} + \Omega\tau_0\ddot{e}) + aT_0(\dot{C} + \gamma\ddot{C}) = K\theta_{,ii}, \tag{4}$$

and the equation of mass diffusion,

$$D\beta_2 e_{,ii} + Da(\theta + \tau_1\dot{\theta})_{,ii} + (\dot{C} + \Omega\tau^0\ddot{C}) - Db(C + \tau^1\dot{C})_{,ii} = 0, \tag{5}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (i, j = 1, 2, 3),$$

$$\beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu)\alpha_c,$$

λ, μ are Lame's constants, α_t is the coefficient of linear thermal expansion, and α_c is the coefficient of linear diffusion expansion. $\theta = T - T_0$, T is the absolute temperature, and T_0 is the temperature of the medium in its natural state assumed to be such that $|\theta/\mathrm{T}_0| < 1$. t_{ij}'s are the components of the stress tensor, u_i's are the components of the displacement vector, ρ is the density assumed to be independent of time, e_{ij}'s are the components of the strain tensor, and $e = e_{kk}$. P is the chemical potential per unit mass, C is the concentration, C_E is the specific heat at constant strain, and K is the coefficient of thermal conductivity; D is the thermoelastic diffusion constant. τ_0, τ_1 are the thermal relaxation times, which ensure that the heat conduction equation, satisfied by the temperature θ, will predict finite speeds of heat propagation. τ^0, τ^1 are the diffusion relaxation times which ensure that the equation, satisfied by the concentration C, will also predict finite speeds of propagation of matter from one medium to another. The constants a and b are measures of thermoelastic diffusion effects and diffusive effects, respectively. The superposed dots denote derivatives with respect to time. δ_{ij} is the Kronecker delta.

For the L–S model, $\tau_1 = 0, \tau^1 = 0, \Omega = 1, \gamma = \tau_0$. The governing equations in the L–S model are the same as given by Sherief et al. [9]. For the G–L model,

$\tau_1 > 0, \tau^1 > 0, \Omega = 0, \gamma = \tau^0$. The thermal relaxation times, τ_0 and τ_1, satisfy the inequality $\tau_1 \geq \tau_0 \geq 0$ for the G–L model. The diffusion relaxation times, τ^0 and τ^1, also satisfy the inequality $\tau^1 \geq \tau^0 \geq 0$ for the G–L model.

3 Formulation and Solution of the Problem

We consider an isotropic homogeneous elastic solid with generalized thermoelastic diffusion in the undeformed state at temperature T_0. We introduce the rectangular Cartesian coordinate system (x, y, z) which has its origin on the surface $z = 0$ with the z-axis pointing normal to the medium. A concentrated, uniformly and linearly distributed normal force or thermal source, or chemical potential source, is assumed to be acting at the origin of the rectangular Cartesian coordinates.

For a two-dimensional problem, we assume

$$\vec{u} = (u_1, 0, u_3) \tag{6}$$

The initial and regularity conditions are given by

$$\begin{aligned} u_1(x, z, 0) &= 0 = \dot{u}_1(x, z, 0), \\ u_3(x, z, 0) &= 0 = \dot{u}_3(x, z, 0), \end{aligned}$$

$$\begin{aligned} &\theta(x, z, 0) = 0 = \dot{\theta}(x, z, 0), \\ &C(x, z, 0) = 0 = \dot{C}(x, z, 0) \\ &P(x, z, 0) = 0 = \dot{P}(x, z, 0) \quad \text{for } z \geq 0, -\infty < x < \infty, \end{aligned} \tag{7}$$

$$\begin{aligned} u_1(x, z, t) &= u_3(x, z, t) = \theta(x, z, t) \\ &= C(x, z, t) = P(x, z, t) = 0 \text{ for } t > 0 \quad \text{when } z \to \infty. \end{aligned} \tag{8}$$

To facilitate the solution, the following dimensionless quantities are introduced:

$$\begin{aligned} &x' = \frac{\omega_1^*}{c_1}x, \quad z' = \frac{\omega_1^*}{c_1}z, \quad t' = \omega_1^* t, \quad u_1' = \frac{\omega_1^*}{c_1}u_1, \quad u_3' = \frac{\omega_1^*}{c_1}u_3, \\ &t_{33}' = \frac{t_{33}}{\beta_1 T_0}, \quad t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad C' = \frac{\beta_2 C}{\rho c_1^2}, \quad \theta' = \frac{\beta_1}{\rho c_1^2}\theta, \quad \gamma' = \omega_1^* \gamma, \\ &\tau_1' = \omega_1^* \tau_1, \quad \tau_0' = \omega_1^* \tau_0, \quad \tau^{1'} = \omega_1^* \tau^1, \quad \tau^{0'} = \omega_1^* \tau^0, \quad P' = \frac{P}{\beta_2}, \\ &m' = \frac{\omega_1^*}{c_1}m. \end{aligned} \tag{9}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_1^* = \frac{\rho C_E c_1^2}{K}.$$

The displacement components, $u_1(x, z, t)$ and $u_3(x, z, t)$, may be written in terms of the potential functions, $\phi(x, z, t)$ and $\psi(x, z, t)$, as

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{10}$$

Using Eqs. 6, 9, and 10, Eqs. 3–5 recast into the following form (after suppressing the primes):

$$\delta \nabla^2 \psi = \frac{\partial^2 \psi}{\partial t^2}, \tag{11}$$

$$\nabla^2 \phi - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \theta - \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = \frac{\partial^2 \phi}{\partial t^2}, \tag{12}$$

$$\nabla^2 \theta = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial \theta}{\partial t} + \varepsilon_1 \left(1 + \Omega \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \nabla^2 \phi + \varepsilon_1 a_1 \left(1 + \gamma \frac{\partial}{\partial t}\right) \frac{\partial C}{\partial t}, \tag{13}$$

$$\nabla^4 \phi + a_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 \theta - \varepsilon_2 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla^2 C + \varepsilon_2 a_2 \left(1 + \Omega \tau^0 \frac{\partial}{\partial t}\right) \frac{\partial C}{\partial t} = 0, \tag{14}$$

where

$$\delta = \frac{\mu}{\lambda + 2\mu}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho C_E (\lambda + 2\mu)}, \quad a_1 = \frac{a(\lambda + 2\mu)}{\beta_1 \beta_2},$$
$$\varepsilon_2 = \frac{b(\lambda + 2\mu)}{\beta_2^2}, \quad a_2 = \frac{1}{bD\eta}, \quad \eta = \frac{\rho C_E}{K}. \tag{15}$$

Applying the Laplace and Fourier transformations defined by

$$\bar{f}(x, z, s) = \int_0^\infty f(x, z, t) \mathrm{e}^{-st} \mathrm{d}t, \tag{16}$$

$$\hat{f}(\xi, z, s) = \int_{-\infty}^\infty \bar{f}(x, z, s) \mathrm{e}^{i\xi x} \mathrm{d}x, \tag{17}$$

to Eqs. 11–14, then eliminating $\hat{\phi}, \hat{\theta}, \hat{C}$, and $\hat{\psi}$ from the resulting expression, we obtain

$$\left(\frac{\mathrm{d}^6}{\mathrm{d}z^6}+Q\frac{\mathrm{d}^4}{\mathrm{d}z^4}+N\frac{\mathrm{d}^2}{\mathrm{d}z^2}+I\right)(\hat{\phi},\hat{\theta},\hat{C})=0, \tag{18}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-\lambda_4^2\right)\hat{\psi}=0, \tag{19}$$

where

$$\begin{aligned}
Q&=\frac{1}{E}[F-3\xi^2E],\\
N&=\frac{1}{E}[G-2F\xi^2+3\xi^4E],\\
I&=\frac{1}{E}[F\xi^4-G\xi^2+H-E\xi^6],\\
\lambda_4^2&=\xi^2+\frac{s^2}{\delta}.
\end{aligned} \tag{20}$$

and

$$\begin{aligned}
E&=[-\varepsilon_2+1](1+\tau^1 s),\\
F&=s\varepsilon_2(1+\tau^1 s)\left[a_2\tfrac{(1+\Omega\tau^0 s)}{(1+\tau^1 s)}+(1+\tau_0 s)+s\right]\\
&\quad+\varepsilon_1 a_1 s(1+\gamma s)(1+\tau_1 s)\,[1+a_1]\\
&\quad+\varepsilon_1 s(1+\tau_1 s)(1+\tau^1 s)(1+\Omega\tau_0 s)\,[\varepsilon_2+a_1]-s(1+\tau_0 s)(1+\tau^1 s)\\
G&=-\left\{\begin{array}{l} s^2\varepsilon_2 a_2(1+\Omega\tau^0 s)\,[(1+\tau_0 s)+s+\varepsilon_1(1+\tau_1 s)(1+\Omega\tau_0 s)]\\ +s^3(1+\tau^1 s)\left[\varepsilon_2(1+\tau_0 s)+\varepsilon_1 a_1^2(1+\gamma s)\right]\end{array}\right\},\\
H&=s^4\varepsilon_2 a_2(1+\tau_0 s)(1+\Omega\tau^0 s).
\end{aligned} \tag{21}$$

The roots of Eq. 18 are $\pm\lambda_l$ $(l=1,2,3)$, and the roots of Eq. 19 are $\pm\lambda_4$. Making use of the radiation condition that $\hat{\phi},\hat{\theta},\hat{C}$, and $\hat{\psi}\to 0$ as $z\to\infty$, the solutions of Eqs. 18 and 19 may be written as

$$\hat{\phi}=A_1\mathrm{e}^{-\lambda_1 z}+A_2\mathrm{e}^{-\lambda_2 z}+A_3\mathrm{e}^{-\lambda_3 z}, \tag{22}$$

$$\hat{\theta}=d_1A_1\mathrm{e}^{-\lambda_1 z}+d_2A_2\mathrm{e}^{-\lambda_2 z}+d_3A_3\mathrm{e}^{-\lambda_3 z}, \tag{23}$$

$$\hat{C}=e_1A_1\mathrm{e}^{-\lambda_1 z}+e_2A_2\mathrm{e}^{-\lambda_2 z}+e_3A_3\mathrm{e}^{-\lambda_3 z}, \tag{24}$$

$$\hat{\psi}=A_4\mathrm{e}^{-\lambda_4 z}, \tag{25}$$

where

$$d_l=\frac{P^*\lambda_l^2+Q^*}{R^*\lambda_l^2+S^*},\quad e_l=\frac{U^*\lambda_l^4+V^*\lambda_l^2+W^*}{X^*\lambda_l^2+T^*},\ (l=1,2,3),$$

$$P^*=\frac{1}{1+\tau^1 s}+\frac{(1+\Omega\tau_0 s)}{a_1(1+\gamma s)},\quad U^*=(1+a_1),$$

$$\begin{aligned}
Q^* &= -\left[\frac{\xi^2+s^2}{1+\tau^1 s}+\frac{\xi^2(1+\Omega\tau_0 s)}{a_1(1+\gamma s)}\right], \quad V^* = -(2(1+a_1)\xi^2+a_1 s^2),\\
R^* &= \frac{1}{\varepsilon_1 a_1 s(1+\gamma s)}, \quad W^* = (1+a_1)\xi^4+a_1 s^2\xi^2,\\
S^* &= -\frac{1}{\varepsilon_1 a_1 s(1+\gamma s)}(\xi^2+(1+\tau_0 s)s)+\frac{1+\tau_1 s}{1+\tau^1 s},\\
X^* &= a_1(1+\tau^1 s)+\varepsilon_2(1+\tau^1 s),\\
T^* &= -\{[a_1(1+\tau^1 s)+\varepsilon_2(1+\tau^1 s)]\xi^2+s\varepsilon_2 a_2(1+\Omega\tau^0 s)\},
\end{aligned} \tag{26}$$

with A_l $(l = 1, 2, 3, 4)$ being arbitrary constants.

4 Applications

4.1 Instantaneous Mechanical Force on the Surface of Half Space

The boundary conditions in this case on the surface $z = 0$ are

$$\begin{aligned}
&(i)\quad t_{33}(x,z,t) = -P_1\psi_1(x)\delta(t), &&(ii)\quad t_{31}(x,z,t) = 0,\\
&(iii)\quad \theta(x,z,t) = 0, &&(iv)\quad P(x,z,t) = 0,
\end{aligned} \tag{27}$$

where $\delta(t)$ is the Dirac delta function, $\psi_1(x)$ specifies a source distribution function along the x-axis, and P_1 is the magnitude of the applied force.

Making use of Eqs. 1, 2, 9, and 10, along with $P_1' = \frac{P_1}{\beta_1 T_0}$ in the boundary conditions (see Eq. 27) and applying the transforms defined by Eqs. 16 and 17 and substituting the values of $\hat{\phi}$, $\hat{\theta}$, $\hat{C}$, $\hat{\psi}$ from Eqs. 22–25 in the resulting equations, we obtain expressions for components of the displacement, stress, temperature distribution, and chemical potential distribution as

$$\hat{u}_1 = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[(-i\xi)(\Delta_1 e^{-\lambda_1 z}-\Delta_2 e^{-\lambda_2 z}+\Delta_3 e^{-\lambda_3 z})+\lambda_4\Delta_4 e^{-\lambda_4 z}]\}, \tag{28}$$

$$\hat{u}_3 = \frac{-1}{\Delta}\{P_1\hat{\psi}_1(\xi)[\lambda_1\Delta_1 e^{-\lambda_1 z}-\lambda_2\Delta_2 e^{-\lambda_2 z}+\lambda_3\Delta_3 e^{-\lambda_3 z}+i\xi\Delta_4 e^{-\lambda_4 z}]\}, \tag{29}$$

$$\hat{t}_{33} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[s_1\Delta_1 e^{-\lambda_1 z}-s_2\Delta_2 e^{-\lambda_2 z}+s_3\Delta_3 e^{-\lambda_3 z}+s_4\Delta_4 e^{-\lambda_4 z}]\}, \tag{30}$$

$$\hat{t}_{31} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[\lambda_1\Delta_1 e^{-\lambda_1 z}-\lambda_2\Delta_2 e^{-\lambda_2 z}+\lambda_3\Delta_3 e^{-\lambda_3 z}-m_1\Delta_4 e^{-\lambda_4 z}]\}, \tag{31}$$

$$\hat{\theta} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[d_1\Delta_1 e^{-\lambda_1 z}-d_2\Delta_2 e^{-\lambda_2 z}+d_3\Delta_3 e^{-\lambda_3 z}]\}, \tag{32}$$

$$\hat{P} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[t_1\Delta_1 e^{-\lambda_1 z}-t_2\Delta_2 e^{-\lambda_2 z}+t_3\Delta_3 e^{-\lambda_3 z}]\}, \tag{33}$$

where

$$\Delta = \{-[s_4\lambda_1 + m_1s_1](d_2t_3 - d_3t_2) + [s_4\lambda_2 + m_1s_2](d_1t_3 - d_3t_1) \\ - [s_4\lambda_3 + m_1s_3] \quad (d_1t_2 - d_2t_1)\}$$

$$\begin{aligned} &\Delta_1 = m_1(d_2t_3 - d_3t_2), \quad \Delta_2 = m_1(d_1t_3 - d_3t_1), \quad \Delta_3 = m_1(d_1t_2 - d_2t_1), \\ &\Delta_4 = [\lambda_1(d_2t_3 - d_3t_2) - \lambda_2(d_1t_3 - d_3t_1) + \lambda_3(d_1t_2 - d_2t_1)], \\ &s_l = b_1\lambda_l^2 - b_1(1 + \tau_1 s)d_l - b_1(1 + \tau^1 s)e_l - b_2 i\xi, \quad (l = 1, 2, 3), \\ &s_4 = (i\xi b_1 + b_2)\lambda_4, \\ &t_l = \xi^2 - \lambda_l^2 + \varepsilon_2(1 + \tau^1 s)e_l - a_1(1 + \tau_1 s)d_l, \end{aligned} \tag{34}$$

4.2 Thermoelastic Interaction Due to Thermal Source

When the plane boundary is free from stress and chemical potential, and is subjected to a thermal point source, the boundary conditions on the surface $z = 0$ are

$$\begin{aligned} &(i) \quad t_{33}(x, z, t) = 0, &&(ii) \quad t_{31}(x, z, t) = 0, \\ &(iii) \quad \theta(x, z, t) = P_3\psi_1(x)\delta(t), &&(iv) \quad P(x, z, t) = 0, \end{aligned} \tag{35}$$

where $\delta(t)$ is the Dirac delta function, $\psi_1(x)$ is the source distribution function along the x-axis, and P_3 is the constant temperature applied on the boundary.

With the use of Eqs. 1, 2, 9, 10, 16, and 17, along with $P_3' = \frac{\beta_1}{\rho c_1^2} P_3$ and boundary conditions of Eq. 35, the corresponding expressions for components of the displacement, stress, temperature distribution, and chemical potential distribution are given by Eqs. 28–33 with Δ_l replaced by Δ_l^* $(l = 1, 2, 3, 4)$ and P_1 replaced by P_3 in Eq. 34, where

$$\begin{aligned} &\Delta_1^* = m_1(s_2t_3 - s_3t_2) + s_4(\lambda_2t_3 - \lambda_3t_2), \\ &\Delta_2^* = m_1(s_1t_3 - s_3t_1) + s_4(\lambda_1t_3 - \lambda_3t_1), \\ &\Delta_3^* = m_1(s_1t_2 - s_2t_1) + s_4(\lambda_1t_2 - \lambda_2t_1), \\ &\Delta_4^* = -[s_1(\lambda_2t_3 - \lambda_3t_2) - s_2(\lambda_1t_3 - \lambda_3t_1) + s_3(\lambda_1t_2 - \lambda_2t_1)], \end{aligned} \tag{36}$$

4.3 Chemical Potential Source on the Surface of Half-Space

Here the plane boundary is stress free, isothermal and is subjected to a chemical potential source; therefore, the boundary conditions on the surface $z = 0$ are

$$\begin{aligned} &(i) \quad t_{33}(x, z, t) = 0, \quad (ii) \quad t_{31}(x, z, t) = 0, \\ &(iii) \quad \theta(x, z, t) = 0, \quad (iv) \quad P(x, z, t) = P_4\psi_1(x)\delta(t), \end{aligned} \tag{37}$$

where $\delta(t)$ is the Dirac delta function, $\psi_1(x)$ is the source distribution function along the x-axis, and P_4 is the constant potential applied on the boundary.

Adopting the same procedure as in the case of the mechanical force and thermal source, along with $P_4' = \frac{P_4}{\beta_2}$ and using the boundary conditions of Eq. 37, the expressions for the components of the displacement, stress, temperature distribution, and chemical potential distribution are given by Eqs. 28–33 by replacing Δ_l with Δ_l^{**} $(l = 1, 2, 3, 4)$ and P_1 with P_4 in Eq. 34 where

$$\begin{aligned}
\Delta_1^{**} &= -[m_1(s_2d_3 - s_3d_2) + s_4(\lambda_2d_3 - \lambda_3d_2),\\
\Delta_2^{**} &= -[m_1(s_1d_3 - s_3d_1) + s_4(\lambda_1d_3 - \lambda_3d_1)],\\
\Delta_3^{**} &= -[m_1(s_1d_2 - s_2d_1) + s_4(\lambda_1d_2 - \lambda_2d_1)],\\
\Delta_4^{**} &= s_1(\lambda_2d_3 - \lambda_3d_2) - s_2(\lambda_1d_3 - \lambda_3d_1) + s_3(\lambda_1d_2 - \lambda_2d_1),
\end{aligned} \tag{38}$$

Case I: Concentrated Normal Force/Thermal Point Source/Chemical Potential Point Source

The solution due to the concentrated normal force/thermal point source/chemical potential point source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x), \tag{39}$$

in Eqs. 27, 35, and 37. Applying the Laplace and Fourier transforms defined by Eqs. 16 and 17 on Eq. 39, gives

$$\hat{\psi}_1(\xi) = 1. \tag{40}$$

Case II: Distributed Sources

(a) Uniformly distributed normal force/uniformly distributed thermal source/uniformly distributed chemical potential source.

The solution due to a uniformly distributed normal force/uniformly distributed thermal source/uniformly distributed chemical potential source applied on the half-space is obtained by setting

$$\psi_1(x) = \begin{cases} 1 \text{ if } |x| \le m, \\ 0 \text{ if } |x| > m, \end{cases} \tag{41}$$

in Eqs. 27, 35, and 37. The Laplace and Fourier transforms with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width $2m$ applied at the origin of the coordinate system $(x = z = 0)$ in dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \left[2 \sin\left(\frac{\xi c_1 m}{\omega_1^*}\right) \Big/ \xi\right], \quad \xi = 0 \tag{42}$$

(b) Linearly distributed normal force/linearly distributed thermal source/linearly distributed chemical potential source.

The solution due to the linearly distributed normal force/linearly distributed thermal source/linearly distributed chemical potential source applied on the half-space surface is obtained by setting

$$\psi_1(x) = \begin{cases} 1 - \frac{|x|}{m} & \text{if } |x| \leq m, \\ 0 & \text{if } |x| > m, \end{cases} \tag{43}$$

in Eqs. 27, 35, and 37 where $2m$ is the width of the strip load. The Laplace and Fourier transform in the case of a linearly distributed normal force/thermal source/potential source applied at the origin on the system in dimensionless form is

$$\hat{\psi}_1(\xi) = \frac{2[1 - \cos(\xi c_1 m/\omega_1^*)]}{\xi^2 c_1 m/\omega_1^*}. \tag{44}$$

The expressions for displacements, stresses, temperature distribution, and chemical potential distribution can be obtained for a concentrated normal force/thermal point source/chemical potential point source, and uniformly and linearly distributed normal forces/thermal sources/chemical potential sources by replacing $\hat{\psi}_1(\xi)$ from Eqs. 40, 42, and 44, respectively, in Eqs. 28–33.

5 Particular Cases

(a) Neglecting the diffusion effect (i.e., $\beta_2 = b = a = 0$), we obtain the corresponding expressions for displacements, stresses, temperature distributions given by Eqs. 28–33 in the thermoelastic half-space as

$$\hat{u}_1 = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[(-i\xi)(-\Delta_1' e^{-\lambda_1 z} + \Delta_2' e^{-\lambda_2 z}) - \lambda_4 \Delta_3' e^{-\lambda_4 z}]\}, \tag{45}$$

$$\hat{u}_3 = \frac{-1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-\lambda_1\Delta_1' e^{-\lambda_1 z} + \lambda_2\Delta_2' e^{-\lambda_2 z} - i\xi\Delta_3' e^{-\lambda_4 z}]\}, \tag{46}$$

$$\hat{t}_{33} = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-s_1^*\Delta_1' e^{-\lambda_1 z} + s_2^*\Delta_2' e^{-\lambda_2 z} - s_4\Delta_3' e^{-\lambda_4 z}]\}, \tag{47}$$

$$\hat{t}_{31} = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-\lambda_1\Delta_1' e^{-\lambda_1 z} + \lambda_2\Delta_2' e^{-\lambda_2 z} + m_1\Delta_3' e^{-\lambda_4 z}]\}, \tag{48}$$

$$\hat{\theta} = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-d_1\Delta_1' e^{-\lambda_1 z} + d_2\Delta_2' e^{-\lambda_2 z}]\}, \tag{49}$$

where

$$\begin{aligned}
&\Delta^* = [s_4(\lambda_1 d_2 - \lambda_2 d_1) + m_1(s_1^* d_2 - s_2^* d_1)],\\
&\Delta_1' = m_1 d_2,\ \Delta_2' = m_1 d_1,\ \Delta_3' = (\lambda_1 d_2 - \lambda_2 d_1),\\
&s_l^* = b_1\lambda_l^2 - b_1(1+\tau_1 s)d_l - b_2 i\xi, \quad (l = 1, 2)\\
&s_4 = (i\xi b_1 + b_2)\lambda_4.
\end{aligned} \tag{50}$$

The above expressions yield the corresponding expressions for the concentrated, uniformly and linearly distributed normal force by replacing $\hat{\psi}_1(\xi)$ from Eqs. 40, 42, and 44, respectively, in Eqs. 45–49.
(b) Making use of the values of $\hat{\psi}_1(\xi)$ from Eqs. 40, 42, and 44 and by replacing Δ_l' with Δ_l'' $(l = 1, 2, 3)$ as given below, we obtain the expressions for displacements, stresses, and temperature distributions in a thermoelastic medium, due to a concentrated, uniformly and linearly distributed thermal point source, where

$$\Delta_1'' = m_1 s_2^* + s_4\lambda_2, \quad \Delta_2'' = m_1 s_1^* + s_4\lambda_1, \quad \Delta_3'' = s_2^*\lambda_1 - s_1^*\lambda_2. \tag{51}$$

6 Special Cases

(a) By putting $\tau_1 = 0, \tau^1 = 0, \Omega = 1, \gamma = \tau_0$ in Eqs. 28–33 and 45–49, we obtain the corresponding expressions of thermoelastic diffusion and thermoelasticity for the L–S theory.
(b) For the G–L theory, we obtain the expressions of thermoelastic diffusion and themoelasticity by substituting $\Omega = 0, \gamma = \tau^0$, and $\tau_1 > 0, \tau^1 > 0$ in Eqs. 28–33 and 45–49.
(c) For the case of coupled thermoelasticity, the thermal relaxation times vanish, i.e., $\tau_0 = \tau^0 = \tau_1 = \tau^1 = 0$ and consequently, we obtain the corresponding expressions of thermoelastic diffusion and thermoelasticity by putting these values in Eqs. 28–33 and 45–49.

7 Inversion of the Transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in Eqs. 28–33 and 45–49 for the L–S, G–L, and C–T theories. These expressions are functions of z, the parameters of Laplace and Fourier transforms s and ξ, respectively, and hence, are of the form $\hat{f}(\xi, z, s)$. To obtain the function $f(x, z, t)$ in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x, z, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-i\xi x}\hat{f}(\xi, z, s)\mathrm{d}\xi = \frac{1}{\pi}\int_{0}^{\infty}(\cos(\xi x) f_{\mathrm{e}} - i\sin(\xi x) f_0)\mathrm{d}\xi, \tag{52}$$

where f_e and f_0 are, respectively, even and odd parts of the function $\hat{f}(\xi, z, s)$. Thus, Eq. 52 gives us the Laplace transform $\bar{f}(x, z, s)$ of the function $f(x, z, t)$. Following Honig and Hirdes [15], the Laplace transform function $\bar{f}(x, z, s)$can be inverted to $f(x, z, t)$.

The last step is to calculate the integral in Eq. 52. The method for evaluating this integral is described by Press et al. [16], which involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8 Numerical Results and Discussion

Following Sherief and Saleh [10], copper material is chosen for the purpose of numerical calculation.

$T_0 = 293\,\text{K}$, $\rho = 8954\ \text{kg}\cdot\text{m}^{-3}$, $C_E = 383.1\ \text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$, $\alpha_t = 1.78 \times 10^{-5}\,\text{K}^{-1}$

$\alpha_c = 1.98 \times 10^{-4}\,\text{m}^3\cdot\text{kg}^{-1}$, $K = 386\,\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$, $\lambda = 7.76 \times 10^{10}\,\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$

$\mu{=}3.86{\times}10^{10}\,\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$, $D = 0.85{\times}10^{-8}\,\text{kg}\cdot\text{s}\cdot\text{m}^{-3}$, $\alpha{=}1.2{\times}10^4\,\text{m}^2\cdot\text{s}^{-2}\cdot\text{K}^{-1}$

$b = 0.9 \times 10^6\,\text{m}^5\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$

The values of the dimensionless relaxation times, $\tau_0 = 0.02$, $\tau^0 = 0.2$ have been taken from Sherief and Saleh [10] and the values of τ_1, τ^1 are taken proportionally of comparable magnitude as $\tau_1 = 0.03$, $\tau^1 = 0.3$.

The values of normal displacement u_3, normal stress t_{33}, temperature distribution θ, and chemical potential distribution P for thermoelastic diffusion (TED) and thermoelasticity (TE) have been studied for a normal force/thermal source/chemical potential source at time $t = 0.5$. The variations of the components with distance x are shown: (a) solid line for TED and solid line with center symbol 'circle' for TE for L–S theory, (b) small dashed line for TED and small dashed line with center symbol 'triangle' for TE for G–L theory, and (c) long dashed line for TED and long dashed line with center symbol 'square' for TE for C–T theory. The variations are shown in Figs. 1–12. The computations are carried out in the range $0 \leq x \leq 10$.

8.1 Mechanical Sources on the Surface of Half-Space

8.1.1 Concentrated Force

Figure 1 shows the variation of normal displacement u_3 with distance x, which for TED has converging oscillatory behavior for L–S, G–L, and C–T theories. The values of normal displacement for TE have similar behavior as TED for both L–S and C–T theories whereas for G–L theory, the values of u_3 increase sharply in the range $0 \leq x \leq 2.5$, $7 \leq x \leq 10$ and decrease outside this range.

Figure 2 shows the variation of normal stress t_{33} with distance x, which for TED has converging oscillatory behavior for the L–S theory whereas it has an oscillatory behavior for both G–L and C–T theories in the range $0 \leq x \leq 10$. The values of

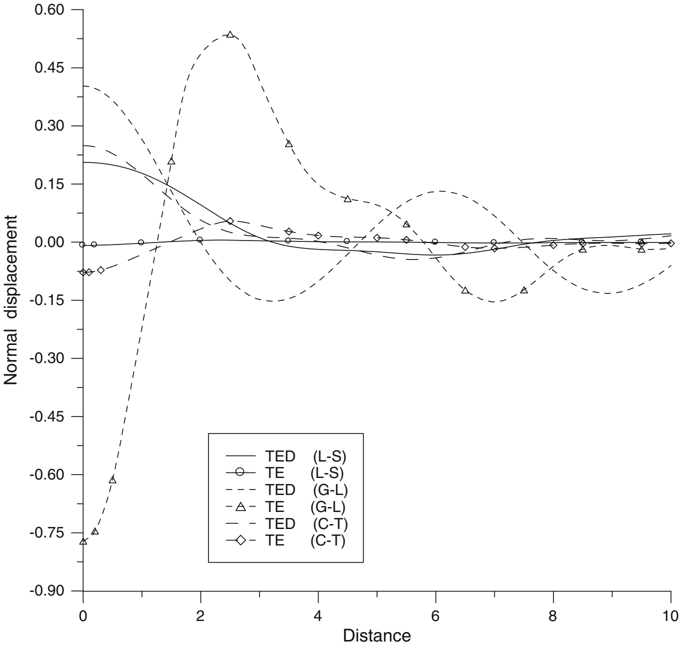

Fig. 1 Variation of normal displacement u_3 with distance x (concentrated normal force)

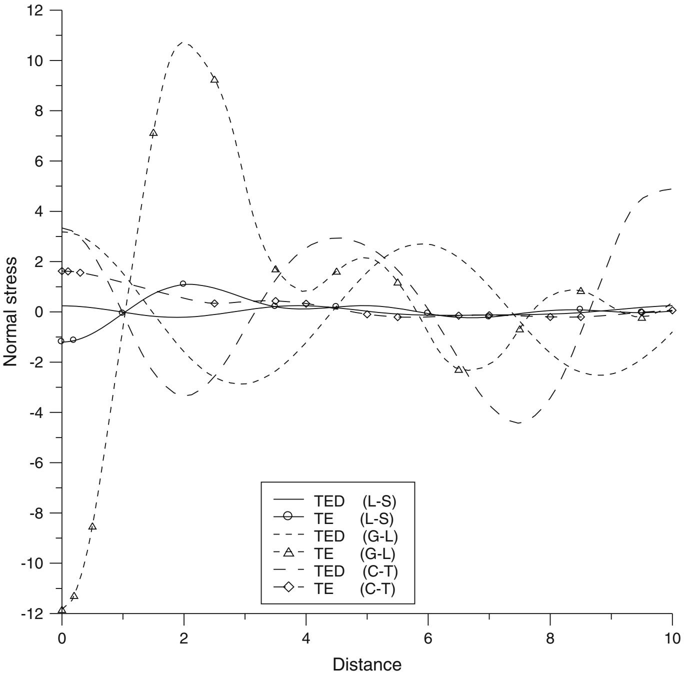

Fig. 2 Variation of normal stress t_{33} with distance x (concentrated normal force)

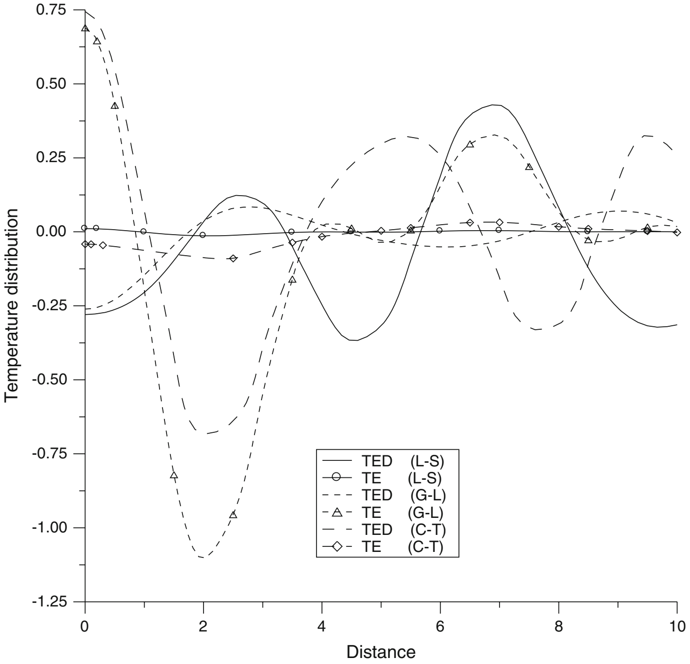

Fig. 3 Variation of temperature distribution θ with distance x (concentrated normal force)

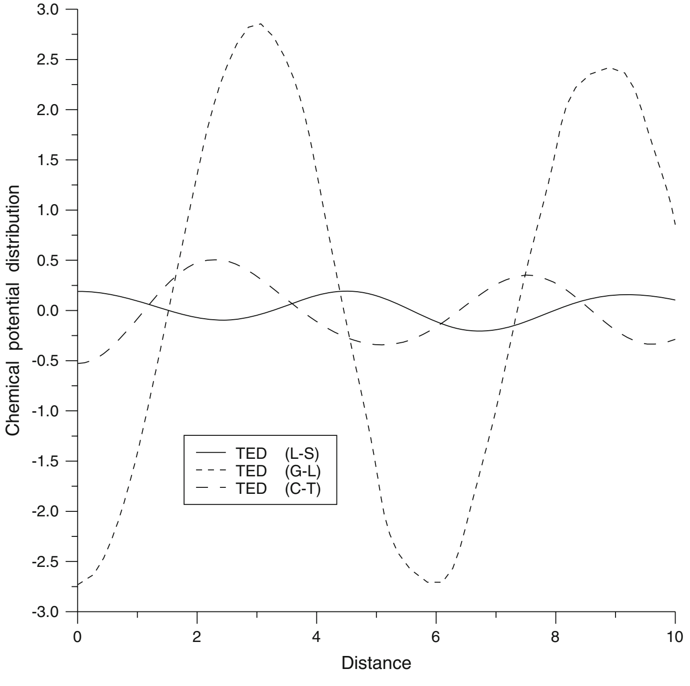

Fig. 4 Variation of chemical potential distribution P with distance x (concentrated normal force)

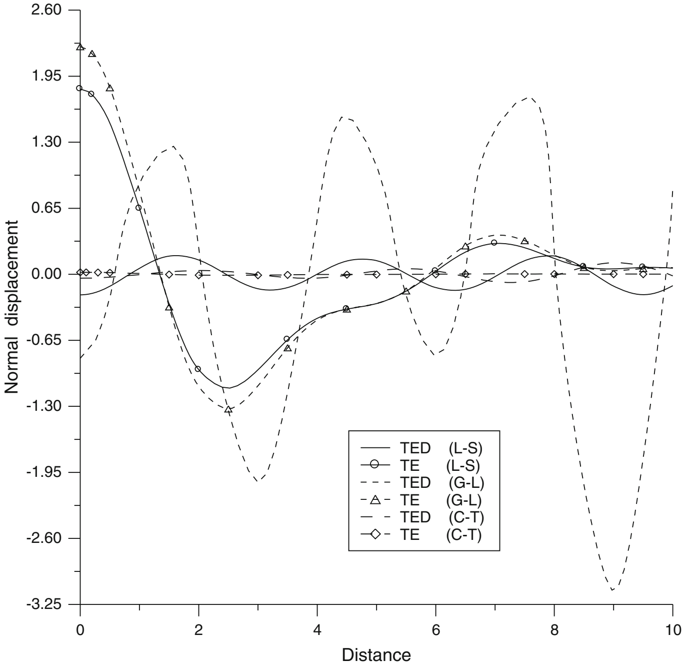

Fig. 5 Variation of normal displacement u_3 with distance x (uniformly distributed thermal source)

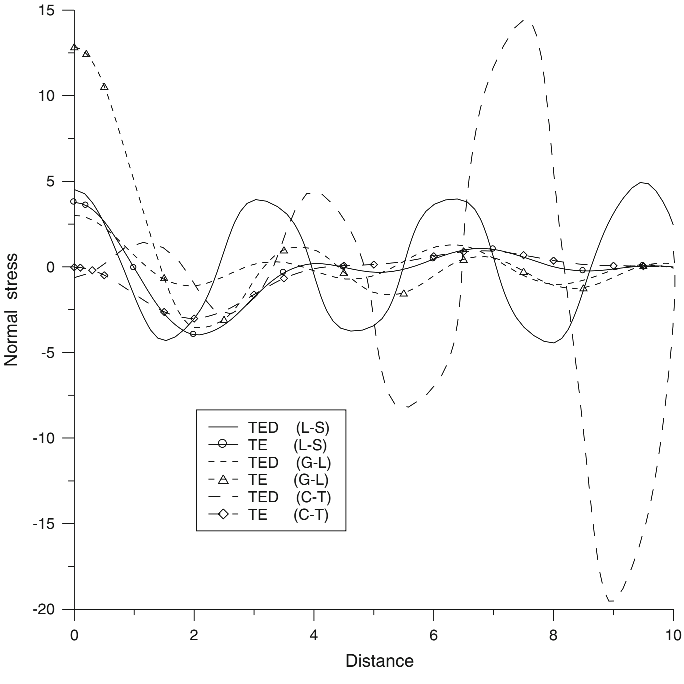

Fig. 6 Variation of normal stress t_{33} with distance x (uniformly distributed thermal source)

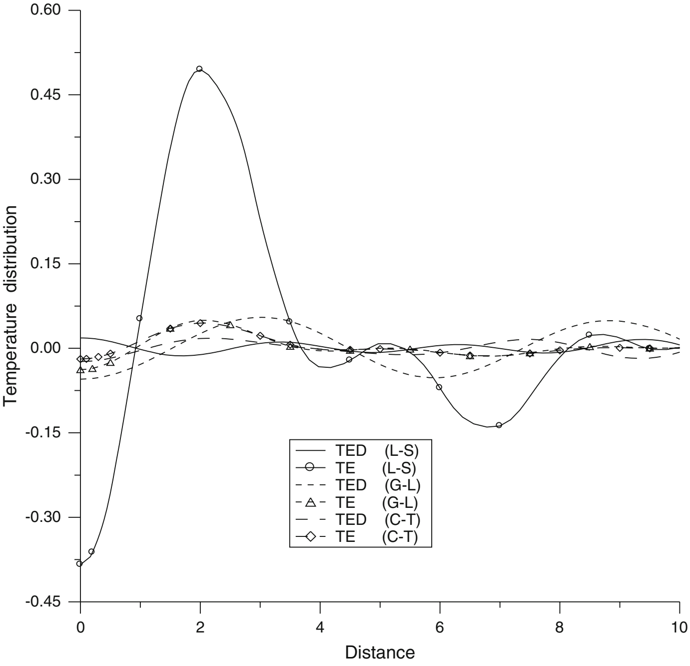

Fig. 7 Variation of temperature distribution θ with distance x (uniformly distributed thermal source)

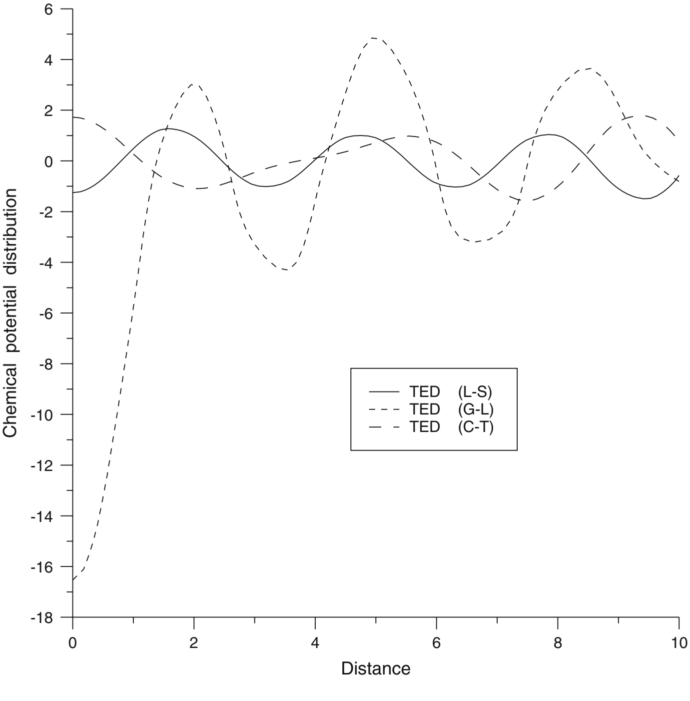

Fig. 8 Variation of chemical potential distribution P with distance x (uniformly distributed thermal source)

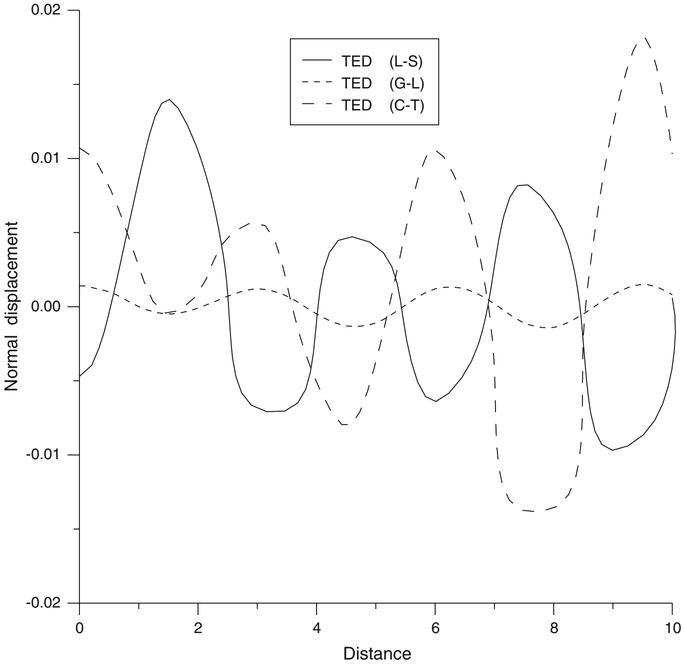

Fig. 9 Variation of normal displacement u_3 with distance x (uniformly distributed chemical potential source)

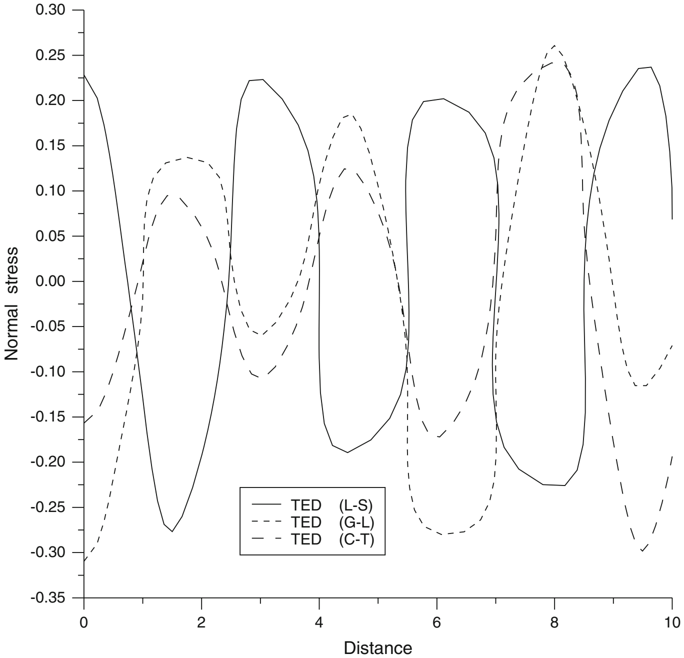

Fig. 10 Variation of normal stress t_{33} with distance x (uniformly distributed chemical potential source)

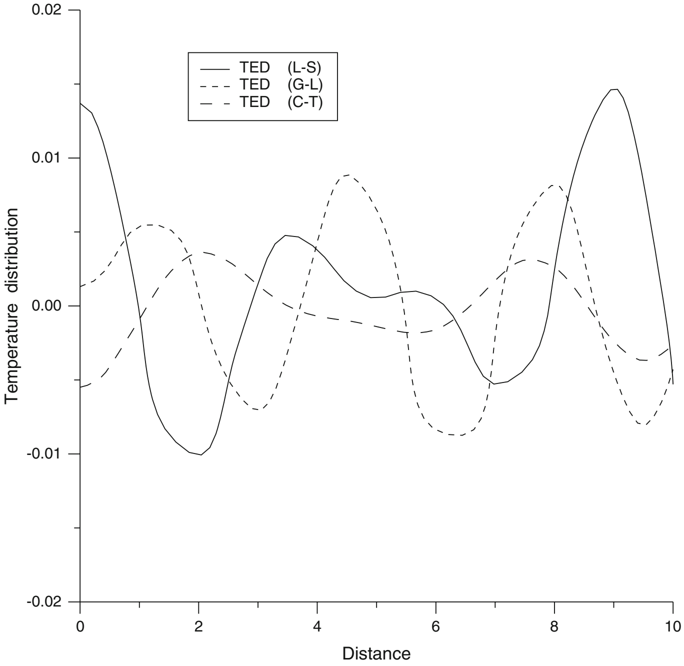

Fig. 11 Variation of temperature distribution θ with distance x (uniformly distributed chemical potential source)

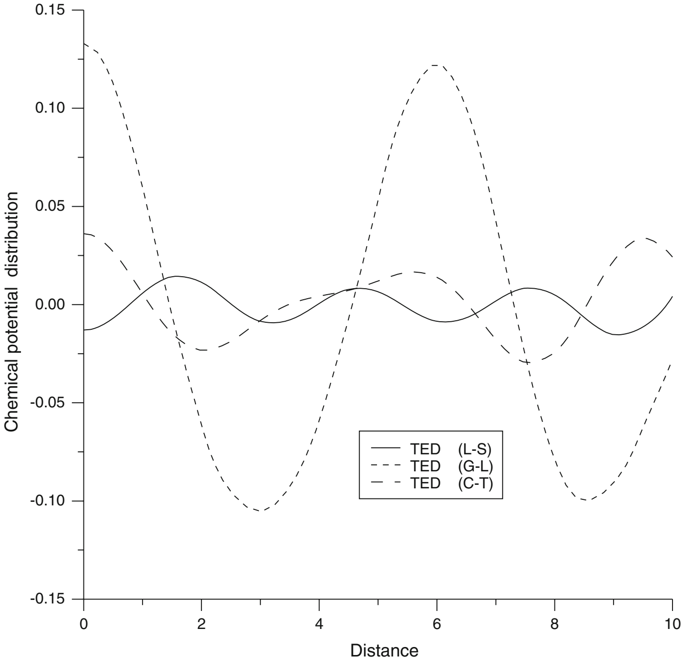

Fig. 12 Variation of chemical potential distribution P with distance x (uniformly distributed chemical potential source)

the normal stress for TE have converging oscillatory behavior for both L–S and G–L theories whereas for the C–T theory, the values of t_{33} increase sharply in the range $0 \leq x \leq 2$, decrease in the range $2.2 \leq x \leq 4$, and as x increases further, have an oscillatory behavior.

Figure 3 shows the variation of the temperature distribution θ with distance x, which for TED has an oscillatory behavior for L–S, G–L, and C–T theories. The values of the temperature distribution for TE are very close to each other for both L–S and C–T theories, but for G–L theory, the values of θ decrease sharply in the range $0 \leq x \leq 2$, increase in the range $2.2 \leq x \leq 4$ and then become very close to L–S and C–T theories.

Figure 4 shows the variation of the chemical potential distribution P with distance x, which for TED has an oscillatory behavior in the range $0 \leq x \leq 10$ for both L–S and C–T theories. The values of the chemical potential increase sharply in the ranges, $0 \leq x \leq 3$ and $0 \leq x \leq 9$, and decrease sharply outside this range for the G–L theory.

8.2 Thermoelastic Interaction Due to Thermal Sources

8.2.1 Uniformly Distributed Thermal Source

Figure 5 shows the variation of the normal displacement u_3 with distance x, which for TED has an oscillatory behavior for L–S, G–L, and C–T theories. The values of the normal displacement for TE are very close to each other for L–S and C–T theories whereas for the G–L theory, the values of u_3 decrease in the range $0 \leq x \leq 3$ and then have an oscillatory behavior.

Figure 6 shows the variation of the normal stress t_{33} with distance x, which for TED has an oscillatory behavior for L–S and G–L theories, whereas for the C–T theory, it has an oscillatory behavior in the range $0 \leq x \leq 5$, increases in the range $5.1 \leq x \leq 7$, $9 \leq x \leq 10$, and decreases in the range $7.1 \leq x \leq 9$. The values of the normal stress for TE have an oscillatory behavior (close to each other) for L–S, G–L, and C–T theories.

Figure 7 shows the variation of the temperature distribution θ with distance x, which for TED has an oscillatory behavior for L–S and C–T theories, whereas for the G–L theory, it increases sharply in the range $0 \leq x \leq 3$, $6 \leq x \leq 8$ and decreases sharply outside this range. The values of the temperature distribution for TE increase in the range $0 \leq x \leq 2$, decrease in the range $2.5 \leq x \leq 4$ and, as x increases, have an oscillatory behavior for L–S, G–L, and C–T theories.

Figure 8 shows the variation of the chemical potential distribution P with distance x, which for TED increases sharply in the range $0 \leq x \leq 2$ and then has an oscillatory behavior for the G–L theory whereas it has an oscillatory behavior in the range $0 \leq x \leq 10$ for both L–S and C–T theories.

8.3 Chemical Potential Source on the Surface of Half-Space

8.3.1 Uniformly Distributed Chemical Potential Source

Figures 9–12 show the variation of the normal displacement u_3, normal stress t_{33}, and temperature distribution with distance x, which for TED has an oscillatory behavior in the range $0 \leq x \leq 10$ for L–S, G–L, and C–T theories, but their magnitudes of oscillation are different.

9 Conclusions

The effect of diffusion plays an important role in the study of the deformation of an elastic body. It is observed that as x diverges from the point of application of a source, the components of normal displacement, stress and chemical potential distribution follow an oscillatory path. The salient feature of these results is that the displacement field, the stresses, and temperature distribution contain new terms characterizing the influence of the thermodiffusion. As the disturbance travels through different constituents of the medium, it encounters sudden changes, resulting in an inconsistent/non-uniform pattern of curves. The trend of curves exhibits the properties of thermal diffusivity of the medium and satisfies the requisite condition of the problem. The results of this problem are very useful in the two-dimensional problem of dynamic response due to various sources of thermoelastic diffusion which have various geophysical and industrial applications.

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